The British
Psychological Society

Activity-oriented teaching of stochastics in elementary school

Ulrike Kipman[1]* , Anton Kühberger[2] and Belinda Pletzer[2]

[1]College of Education, Salzburg, Austria
[2]University of Salzburg, Austria

Background. There is little research on how to best introduce children to stochastics. In general, demonstration and concrete experience seem to be necessary to establish good understanding of stochastics in children. Pupils seem to be able to develop an intuition on stochastic thinking when they actively solve probabilistic problems and carry out probability experiments based on age-adequate content and materials.

Aims. This study investigates how activity-oriented education can improve stochastics achievement of children. We aim to clarify whether and how much stochastics achievement of elementary school children can be improved comparing different teaching methods and we want to identify an instruction method that is most suitable for improving stochastics achievement of elementary school children.

Sample. A total of 617 primary and secondary school children aged 6–12 years participated in the study. Of those, 324 children had student-centred activity-oriented education in stochastics between the two tests, 202 children were instructed by a teaching approach where worksheets were used and 91 children with no specific intervention served as control group.

Method. We analysed gain in stochastics performance for each group using a MLM and several repeated measures ANOVAs.

Results. All three groups improved their performance from the first to the second test session. Independent of social background and gender – student-centred activity-oriented education was more effective in improving test scores in all fields of stochastics than the rather teacher-centred approach where worksheets were used or no specific stochastics education.

Conclusion. We conclude that children can improve their understanding of stochastics considerably during the elementary school years and that teaching via hands-on experience is more successful than a teaching approach where worksheets are used.

Children are often confronted with stochastics. They are required to make probabilistic decisions, to weigh risks and take chances (Paparistodemou & Philipou, 2002), not only when they are playing games, but also in daily life (e.g., when being unsure in multiple choice – tests at school; when deciding which way to go when they have lost their way). The term stochastics contains (1) combinatorics, (2) probability theory, and (3) statistics (Lehner & Mehltretter, 2009). Therefore, dealing with tables and graphs is also part of

*Correspondence should be addressed to Ulrike Kipman, Pädagogische Hochschule Stefan Zweig, College of Education, Akademiestr. 23, Salzburg 5020, Austria (email: Ulrike.Kipman@phsalzburg.at).

stochastics (by belonging to statistics which itself belongs to stochastics), as is combinatorics and probability theory.

How early and to what extent children can and should be taught stochastics has been discussed for a long time. The importance of teaching stochastics early has been recognized by the US National Council of Teachers of Mathematics (2000) in their recommendation to incorporate the teaching of probability and randomness in elementary school curricula (Langrall & Mooney, 2005). Providing basic stochastic tools in elementary school is also recommended in Europe (e.g., Martignon & Krauss, 2009).

When planning this study, we checked for stochastics in the curricula of Austrian and Bavarian children. We found that reading and drawing graphs and tables (as part of the 'statistical education') as well as conducting simple random experiments (as part of 'stochastics education/probability') is part of the Austrian and the Bavarian curriculum at the end of grade 4. Higher stochastics education in terms of combinatorics and probability theory is implemented late in these curricula. We find higher stochastics education in the curricula for higher secondary school, for example in Austria in grade 10 (random experiments, probability theory), grade 11 (mean, variance, discrete distributions), and grade 12 (testing hypotheses). In Bavaria, the curriculum contains these issues for grades 10–12 (interpreting statistical data, describing randomness, conditional probability, combinatorics) (Curriculum Mathematics for Primary School in Austria, 2007; Curriculum Mathematics for Primary School in Bavaria, 2014; Curriculum Mathematics for Secondary School in Austria, 2014; Curriculum Mathematics for Secondary School in Bavaria, 2014).

Back to the question how early and to what extent children can and should be taught stochastics: Piaget and Inhelder (1975) state that children begin to understand probabilities – the core concept of stochastics – around the age of seven. Since then, several authors have suggested that children are able to intuitively solve probability tasks much earlier (e.g., Hodnik-Cadez & Skrbec, 2011; Mousalides & English, 2009; Nikiforidou & Pange, 2007, 2009). However, there is a large variety of tasks employed in these studies (Langrall & Mooney, 2005), sample sizes are usually small (e.g., Nikiforidou & Pange, 2009), and several studies report only qualitative findings (e.g., Mousalides & English, 2009). Thus, little consensus exists about the age at which children begin to understand probability (Langrall & Mooney, 2005).

Furthermore, there is little research and consensus on the problem of teaching.

Bosma and Resing (2006), as well as Kohnstamm (2014), state that children should have an active role when learning about new topics. Gage and Spiegelhalter (2016) suggest hands-on activities to develop skills and a conceptual understanding of probability. Martignon and Krauss (2009) assume that learning stochastics based on hands-on activities constitute a successful step towards problemsolving. Several studies show better learning output when children get activity-based training (Resing, 2013; Resing, Tunteler, & Elliott, 2015; Tunteler, Pronk, & Resing, 2008), and when children are getting feedback from their teacher (Elliott, Grigorenko, & Resing, 2010; Grigorenko & Sternberg, 1998; Haywood & Lidz, 2007). Butts, Hoffman, and Anderson (1993) also found that activity-based learning has the best outputs when children form their own ideas and hypotheses before experimenting.

These arguments comply well with a constructivist learning perspective. Constructivism views learners as active constructors of their own knowledge and proposes as its first principle that understanding results from active interactions with the environment

(e.g., Savery & Duffy, 1995). The constructivist view is utilized by activity-oriented education (Schneider, 2011; Taber, 2011).

To the best of our knowledge, no study has yet systematically investigated the various facets of stochastics achievement in elementary school children. In addition, no study has directly contrasted activity-oriented education with a teaching approach where worksheets are used to explain stochastics in elementary school. The aim of this study is manifold: First, we aim to clarify whether and how much stochastics achievement of elementary school children can be improved. Second, we wish to identify the instruction method that is most suitable for improving stochastics achievement of elementary school children: (student-centred) activity-oriented education, or a teaching approach applying worksheets. Third, we investigate whether different facets of stochastic thinking improve in a comparable fashion. Fourth, we explore the influence of age, sex, and social background of the children, that is, whether boys or girls, and younger or older children improve differently.

To address these questions, stochastic achievement was assessed before and after one of three interventions in a large sample of 617 children of grades 1–4. For the detailed assessment of stochastics achievement, we developed a quiz distinguishing between the children's ability to understand what we take to be core concepts of stochastics. We used two treatment groups and one control group. Control group children received no specific stochastics education. One experimental group received a teaching approach based on worksheets, the second experimental group received activity-oriented education with materials specifically designed to provide hands-on experience.

Method

Participants

A total of 617 primary and secondary school children (310 boys, 307 girls) aged 6–12 years (mean age: 8.64 ± 1.14 years) participated in the study. All parents gave their informed written consent for their child's participation in the study.

A total of 51 teachers (19 men, 32 women, age: 24–55 years) were trained in teaching with the activity-oriented materials, in courses held by the first author (courses took 4×45 min). Their training was supported by videos[1] (see Appendix). They received detailed written instructions and the KI(D)S-case. Of the 51 teachers, 18 were experienced, the remaining 33 were in education and did apply the activity-oriented materials in groups of two or three in the presence of their supervisor in the classroom. Another 28 teachers (10 men, 18 women, age: 24–46 years; 24 in education and 4 experienced teachers) were trained in teaching with the worksheets (courses also took 4×45 min). Finally, 21 experienced teachers did the teaching in the control group (3 men, 18 women, age: 24–41 years). Teachers of all groups were invited to solve the quiz on their own to get familiar with the tasks.

Material

Stochastics achievement – the KI(D)S-Quiz

The KI(D)S-Quiz is a reliable 46-item instrument, available in two forms (A and B) with a Cronbach α of .88 (Kipman, Fritz, Pletzer, & Kühberger, 2014). The quiz consists of five

---

[1] *Videos are in German because the project included only German speaking teachers and children.*

subtests: (1) terms, (2) tables, (3) graphs, (4) random experiments, and (5) combinatorics (described in more detail below) and a demographic part containing information about *age, grade* (mathematical experience), g*ender, social background, mathematical ability, reading ability*, and *mathematical interest*. Children were also asked whether they 'like mathematics' and 'what grade they got in mathematics', but we did not use grades and self-ratings of interest for our analyses since grades in mathematics and student's self-rated interest of mathematics showed a rather right-skewed distribution (48% had grade 1, 34% grade 2, and 77% stated to like mathematics). The predictive power of actual mathematical ability and mathematical interest in the self-rating thus was low, and we based our analyses on teacher ratings of childrens' mathematical interest and mathematical abilities (scale from 1 to 5). Importantly, mathematical ability was rated in reference to students of the same grade. Teachers also rated *reading skills* of students on a scale of 1–5. Social background was assessed by having pupils indicating the number of books in their home. This variable is a good predictor of socioeconomic status (Bos *et al.*, 2003). Due to the lack of student's familiarity with the concept of stochastics, interest in stochastics was not assessed.

The KI(D)S-Quiz[2] had to be completed during a regular mathematics lesson of 45 min. Each subtest required a comparable amount of time. The scoring of the different tasks reflects three objectives: (1) to be responsive to task difficulty (more points for more difficult items), (2) to minimize the influence of guessing (by earning points in multiple choice items only if all items of the same type are solved correctly), and (3) to weigh the five subtests equally. The total test score is the sum of the five subtest scores (12 points each), reaching a maximal score of 60 points.

The KI(D)S-Quiz consists of five subtests. Examples for each subtest are given below.

1. *Terms*: A total of 16 items – four items each block – assesses children's understanding of the terms 'possible', 'certain', 'impossible', and 'probable'. Children are shown four boxes filled with coloured marbles and they have to answer four questions on each box by checking 'right' or 'wrong' (e.g., 'It is possible to draw a black marble?').
2. *Graphs*: Two items evaluate children's ability to match a text and a bar chart out of three possibilities (e.g., 'There are 15 boys and 8 girls in a classroom'). In addition, children answer four questions about a given bar chart.
3. *Tables*: First, children have to fill in a three-column table by counting five activities of a fictional character in a story. Then, children are presented with a table and have to answer three questions concerning the table.
4. *Random Experiments*: Children are shown three wheels of fortune and have to identify the most likely outcome (black or white), and they are shown a game of dice between two children betting on different outcomes and have to decide who most likely was going to win.
5. *Combinatorics*: Children have to enumerate all possible outcomes of a drawing: twice with replacement, twice without replacement, and once in an 'n over k' – task (e.g., 'An ice-cream parlor sells 4 kinds of ice-cream: chocolate, vanilla, strawberry and nut. You want to buy 2 scoops and don't want to buy one kind twice, how many possibilities do you have?').

---

[2] *The KI(D)S-Quiz is available on request from the first author.*

Procedure

All children completed the KI(D)S-Quiz twice, once before intervention, and once after 8 weeks of intervention. Classes with 15 children on average were randomly assigned to one of three groups: a control group receiving no specific stochastics education,[3] and two intervention groups, receiving 8 weeks of teacher-centred education by worksheets, or of activity-oriented (student-centred) education (see below). The three groups did not differ significantly in any of the control variables before training. Thus, they were comparable in respect to gender, age, social background (SES), grade in mathematics, teacher ratings of mathematical skills, reading skills, and mathematical interest. In each group, about half of the children received Form A of the quiz during the first session and Form B during the second session, the other half received the reversed order, with no effects on performance. Table 1 reports these findings in detail.

Children in the student-centred activity-oriented group learned with materials from the KI(D)S-case. The KI(D)S-case was specifically developed for activity-oriented education in stochastics. It is a patented learning containing wheels of fortune, dices, coloured dices, big plastic letters, plug-cubes, playing cards, calculation sticks, and plastic animals. Children participated in eight lessons (45 min each, two concerning *terms*, one *graphs*, one *tables*, two *combinatorics,* and two concerning *random experiments*) of activity-oriented stochastics education by their regular math teachers based on the demo videos (see Appendix) demonstrating the use of the KI(D)S materials. After initial verbal input from the teachers, they were invited to solve stochastic problems on their own in groups of four to six children. Children could practice stochastics with the provided materials and run their own random experiments. In the *combinatorics* lesson for example, children investigated the number of different possibilities of arranging three animals in a row. In the *random experiments* lessons, children throw dices to find the probabilities of throwing a single 6 or a pair of 6. In addition, children learned about the meaning of *terms*: (certain, possible, impossible, probable) how to interpret *graphs* and how to read a *table*. Examples in the lessons were different from the examples used in the KI(D)S-Quiz.

Purpose was that children would vary strategies, build and test hypotheses, make observations concerning the questions, draw comparisons, and discuss approaches and their solutions. Teachers observed the children, showed possible strategies and suitable heuristics and provided constructive feedback. The aim of the intervention was to enable children to make their own hands-on experiences and to systematically solve stochastics issues (estimate probabilities using heuristics, solving combinatoric tasks like variation/combination – tasks and find out the difference between combination, variation, and permutation). This way of teaching conforms to the idea of activity-oriented education, which is characterized by providing the possibility for pupils to 'newly combine' and 'post-create' things to improve understanding.

Children in the worksheet group received – after the same initial verbal input, like children in the student-centred group – eight worksheets containing examples and tasks for the topics assessed by the KI(D)S-Quiz: *terms*, *combinatorics*, *random experiments*, *tables,* and *graphs*. To ensure comparability between the activity-oriented (student-centred) and the teaching approach where worksheets are used, the worksheet tasks were comparable or identical to the tasks in KI(D)S-case. Worksheets were taken from

---

[3] *As reading and interpreting tables and graphs was not in the curriculum of these children before this study, we assume that all children had no specific previous knowledge. Analyses also showed that groups were comparable before intervention and that there was no ceiling effect solving the tables and graphs part of the quiz.*

Table 1. Demographic data of the three groups

	No education	Worksheet teaching approach	Activity-oriented (student-centred) education	Comparison
N	91	202	324	
Gender[a]				
Boys	45 (50%)	96 (48%)	169 (52%)	$X^2 = 1.10, p = .58$
Girls	46 (50%)	106 (52%)	155 (48%)	
Age[b] (Mean $\pm$ SD)	8.56 $\pm$ 1.10	8.53 $\pm$ 1.14	8.73 $\pm$ 1.15	$F(2) = 2.13, p = .12$
SES[c,d] (Median)	3	3	3	$X^2 = 2.78, p = .25$
Order KI(D)S[a]				
A–B	41 (45%)	93 (46%)	136 (42%)	$X^2 = 0.91, p = .64$
B–A	50 (55%)	109 (54%)	188 (58%)	
Math skills[c,e] (Median)	2	2	2	$X^2 = 3.76, p = .15$
Math interest[c,e] (Median)	2	2	2	$X^2 = 6.16, p = .05$
Reading skills[c,e] (Median)	2	2	2	$X^2 = 1.71, p = .43$

Notes. [a]Nominal, chi-square test.
[b]Scale, one-way-ANOVA.
[c]Ordinal, Kruskal–Wallis test.
[d]Student ratings (0–5).
[e]Teacher ratings (1–5).

Lehner and Mehltretter (2009), a textbook intended to provide material for the teaching of stochastics to elementary school children. As for activity-oriented education, 45 min were reserved to solving the worksheet tasks.

Children in the control group did not receive any focused education in stochastics. The teachers did not explain the items of the KI(D)S-Quiz to them and they did not receive any materials related to the quiz. They were, however, able to acquire knowledge on their own, by asking their parents, discussions with peers or experimenting with games or toys, after receiving the first quiz.

Results

Multi-level analysis
Because of students nesting in classes,[4] we ran a two-level analysis. ICC was low: Less than 6% of variance was explained on school-level and vice versa more than 93% of variance was explained on individual level. This means, teacher did not have a notable impact on the increase in stochastics achievement.

Stochastics achievement before intervention
To evaluate whether pre-intervention differences existed on performance in general and in subtests, a 6 × 3 MANOVA with subtest (*terms, graphs, tables, combinatorics, random experiments, overall test score*) as within-subjects factor, and group as between-subjects factor was performed on the scores achieved before training.

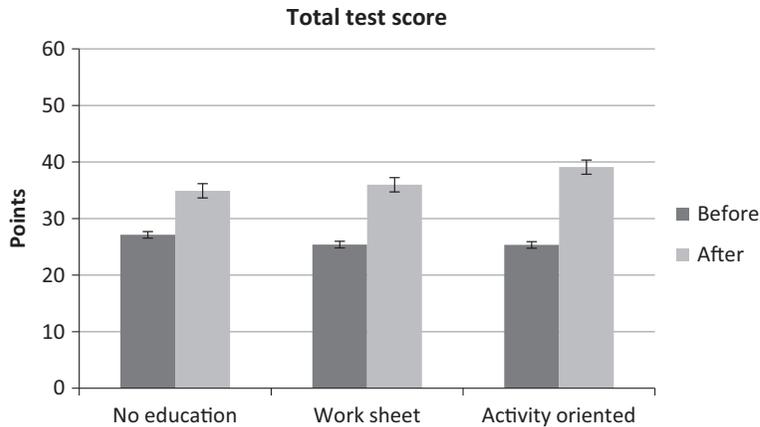[4] *We did not have more than one class per school.*

Figure 1. Total test score of stochastics performance before and after one of three interventions. Error bars represent standard errors.

There was no significant difference between groups in overall test score, $F(2) = 2.67$, $p = .70$, $\eta^2 = .010$.

A *post-hoc* Sidak test indicated that scores in c*ombinatorics* were significantly lower than scores in all other subtests (all $p < .001$), while scores in *graphs* and *tables* were significantly higher than in the other three subtests (all $p < .001$).

Learning due to intervention
The overall results before and after intervention for all groups are depicted in Figure 1. To investigate the gain in performance in an overall analysis, we did a 2×3 repeated measures ANOVA with session as within-subjects factor and group as between-subjects factor for the overall test score and for each subtest.

Activity-oriented (student-centred) education
There was an impressive overall increase in performance, $F(1, 323) = 467.53, p < .001$, $\eta^2 = .591$. This effect remained significant when including age and gender as additional between-subjects variables, $F(1, 306) = 147.65, p < .001, \eta^2 = .325$. For the subtests, there was a significant increase in the activity-oriented group for *terms*, $F(1, 290) = 79.16, p < .001, \eta^2 = .151$, *graphs*, $F(1, 290) = 57.78, p < .001, \eta^2 = .166$, *tables*, $F(1, 290) = 23.79, p < .001, \eta^2 = .076$, *random experiments*, $F(1, 290) = 675.40, p < .001, \eta^2 = .700$, and *combinatorics*, $F(1, 290) = 628.20, p < .001, \eta^2 = .684$.

Teaching using worksheets
Again, we found increased performance overall, $F(1, 201) = 175.21, p < .001, \eta^2 = .437$. The main effect remained significant when including age and gender as additional between-subjects variables, $F(1, 188) = 76.82, p < .001, \eta^2 = .290$. We also found significant main effects for *terms*, $F(1, 178) = 31.57, p < .001, \eta^2 = .151$, *graphs*, $F(1, 178) = 10.72, p < .001, \eta^2 = .057$, *tables*, $F(1, 178) = 22.38, p = .008, \eta^2 = .112$,

random experiments, F(1, 178) = 263.18, *p* < .001, η^2 = .596, and *combinatorics, F*(1, 178) = 162.41, *p* < .001, η^2 = .472.

No specific stochastics education

The overall increase was significant, *F*(1, 90) = 69.91, *p* < .001, η^2 = .437. Significant increases were found for *tables, F*(1, 81) = 8.89, *p* = .004, η^2 = .099, *random experiments, F*(1, 81) = 65.04, *p* < .001, η^2 = .445, and *combinatorics, F*(1, 81) = 74.13, *p* < .001, η^2 = .478, but not for *terms, F*(1, 81) = 0.546, *p* = .462, η^2 = .007, and not for *graphs, F*(1, 81) = 1.65, *p* = .203, η^2 = .020.

Subtest analysis

We performed a 5 × 2 × 3 repeated measures ANOVA with subtest and session as within-subjects factors and group as between-subjects factor.

As expected, there was a significant main effect of session, *F*(1, 549) = 10.94, *p* < .001, η^2 = .020, indicating that test scores improved significantly from the first to the second test session. Session interacted significantly with subtest, *F*(4, 546) = 506.18, *p* < .001, η^2 = .788, and with group, *F*(2, 549) = 5.60, *p* = .004, η^2 = .020.

The group × session interaction is focal for our analysis. To resolve this, we did two separate 5 × 2 × 2 ANOVAs with subtest and session as within-subjects factors and group as between-subjects factor. In these ANOVAs, the 'worksheet – teaching approach' was compared to no specific education and activity-oriented education was compared to the teaching approach using worksheets. In both comparisons, a significant interaction of session × group was observed, worksheet versus no specific education: *F*(4, 256) = 215.78, *p* < .001, η^2 = .771; activity-oriented student-centred education versus rather teacher-centred approach where worksheets were used: *F*(1, 465) = 558.24, *p* < .01, η^2 = .828. Thus, performance improved more due to student-centred activity-oriented education than due to the teaching approach using worksheets and more due to the teaching approach using worksheets than in the control group.

A significant interaction of session × group, indicating stronger improvement with activity-oriented education compared to the teaching approach using worksheets or no specific education, was confirmed for the subtests *terms, graphs, random experiments*, and *combinatorics*, all *F*(2) > 3.29, all *p* < .05, all η^2 > .012, but not for *tables, F*(2) = 0.401, *p* = .670, η^2 = .001.

In sum, except for the subtest *tables*, which showed comparable increase for all three groups, activity-oriented education was more effective than teaching using worksheets, which was more effective than a control condition of no specific education in elementary school children's performance in stochastics.

The results for subtests are depicted in Figure 2.

Gender and stochastics achievement

The results for gender are depicted in Figure 3. There was no effect of gender on the test scores, *F*(1) = 0.030, *p* = .863, η^2 < .001, indicating that boys and girls did not differ in their stochastics achievement before intervention. However, gender interacted with subtest, *F*(4, 547) = 3.34, *p* = .010, as girls had significantly higher test scores than boys in *tables,* 6.85 ± 3.02 versus 6.09 ± 3.43, *t*(550) = 2.76, *p* < .05. In sum, boys and girls did not differ reliably in their stochastics achievement before intervention.
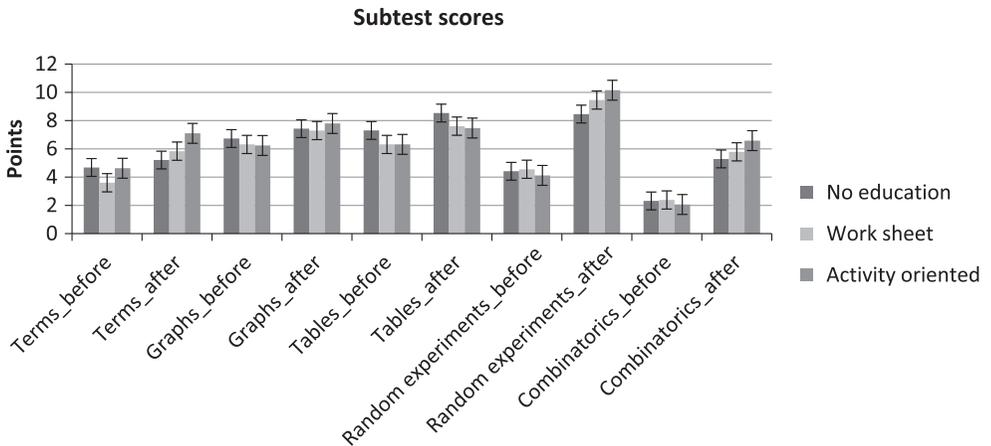
Subtest scores

Figure 2. Performance on each subtest before and after one of three interventions. Error bars represent standard errors.

Total test score - boys
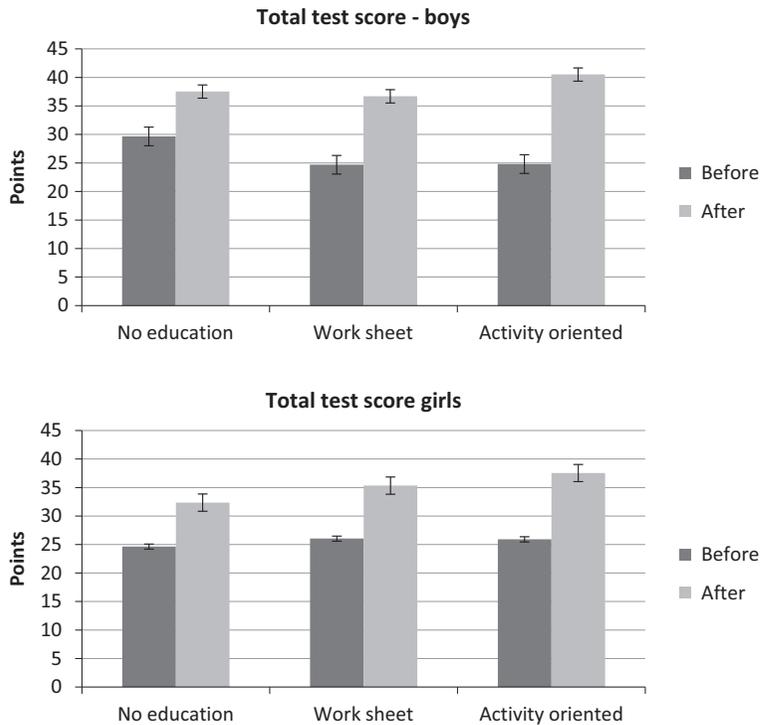

Total test score girls

Figure 3. Overall test score before and after intervention in boys and girls receiving either no education, a teaching approach where worksheets are used, or an activity-oriented teaching approach. Error bars represent standard errors.

To investigate whether boy or girls were influenced differently by the intervention, we ran three separate $5 \times 2 \times 2$ ANOVAs on subtest scores for each group with subtest and session as within-subject factors and gender as between-subjects factor. In the group that

had received no specific education, sex did not interact with either subtest or session or the subtest $\times$ session interaction (all $F < 1.48$, all $p > .216$, all $\eta^2 < .072$), indicating that boys and girls profited equally from repeating the test without intervention on each subtest. In the group teached by worksheets was no interaction between gender and session, $F(1, 177) = 2.66$, $p = .105$, $\eta^2 = .015$, but a significant interaction between gender and subtest, $F(4, 174) = 3.98$, $p = .004$, $\eta^2 = .084$. In the group receiving activity-oriented education, there was a significant interaction between gender and session, $F(1, 289) = 9.76$, $p = .002$, $\eta^2 = .033$, and a significant interaction between gender and subtest, $F(4, 286) = 4.12$, $p = .003$, $\eta^2 = .028$. Boys increased their test scores more strongly after activity-oriented education than girls.

As gender did interact significantly with subtest in both experimental groups, separate 2×2 ANOVAs were run for each subtest. We found significant main effects for *terms, graphs, tables, random experiments,* and *combinatorics* in both intervention groups, all $F(1, 177) > 10.74$, all $p < .001$, all $\eta^2 > .057$ and $F(1, 289) > 23.94$, all $p < .001$, all $\eta^2 > .074$, and a significant interaction between session and gender for the *tables* subtest for the worksheet group, $F(1, 177) = 5.56$, $p = .019$, $\eta^2 = .030$, and for *tables, random experiments,* and *combinatorics* for the activity-oriented group, $F(1, 289) > 4.57$, $p < .033$, $\eta^2 > .015$, indicating a stronger improvement in boys than in girls for these subtests. In summary, whereas performance improved about equally in boys and girls in the group receiving no education, boys seem to have improved more than girls from education. This difference in improvement was especially strong in activity-oriented education.

Age and stochastics achievement

Age is obviously an important variable for stochastics achievement. Before intervention, age was positively correlated with the total test score ($r = .18$, $p < .001$). For subtests, we found a significant correlation for *terms* ($r = .18$, $p < .001$) and *graphs* ($r = .29$, $p < .001$). Improvement in both intervention groups was highest for children aged 6 and 7 years (14.86 points in the rather teacher-centred worksheet group and 19.11 points in the student-centred activity-oriented group) and lowest for children aged 10 years (8.63 points in the rather teacher-centred worksheet group and 12.28 points in the student-centred activity-oriented group).

Social background and stochastics achievement

Social background was not related to the overall stochastics performance before education ($r = .05$, $p = .228$). For subtests, we only found a positive and significant correlation for *graphs* ($r = .12$, $p = .005$). We did not find an interaction between social background and session in the activity-oriented group and in the no education group, $F < .964$, $p < .45$, all $\eta^2 < .054$, in the worksheet group, interaction was significant but had small effect, $F = 2.49$, $p = .033$, $\eta^2 = .061$. Thus, we conclude that social background had no practically relevant impact on any of our measures.

Discussion and conclusion

Our study suggests that children are highly susceptible to stochastics education at elementary school age, irrespective of their social background. Children of all age groups

(7–12 years), even those receiving no specific education, significantly improved in stochastics performance from the first to the second test session, separated by a time interval of 8 weeks. Performance improvement was different for subtests but was strongest after student-centred activity-oriented education in all subtests. Boys and younger children showed bigger increases in student-centred activity-oriented education than did girls and older children, while social background had no impact on performance or improvement.

Improving stochastics understanding – a vote for implementing stochastics education in elementary school curricula

One important lesson learned here is that stochastic performance improved quite remarkably not only in children receiving education, but also in children not receiving any specific education at all. Although these children had no worksheets or hands-on materials available to improve their stochastic skills, they were able to improve considerably within 8 weeks. This would be rather good news, could we be sure that this was due to natural development. Our finding is of course limited because of the small group of children in the control group. In addition, the improvement in the group receiving no specific education did not interact with age, thus it is unlikely that it is simply attributable to cognitive development, or to the possibility that our testing, by incidence, was performed in a sensible period. Note that we were unable to control whether and how extensively children discussed the problems of the KI(D)S-Quiz with their parents or peers, or even experimented with their own materials, for example toys, to understand the questions. Maybe children by mere interest took measures on their own to improve their understanding of test problems they had struggled with during the first test session. But even if children did actually educate themselves, the implications of this observation are twofold and positive: First, they show a remarkable interest in the problems presented in the KI(D)S-Quiz; and second, they are quite successful in improving their stochastics understanding. Taken together, these considerations strongly support the recommendations to include stochastics in elementary school curricula. Building upon their intuitions and their interest, children may acquire a quite advanced understanding of stochastics, which they can build upon later in life.

Improvement after student–centred activity-oriented education – a vote for teaching via hands-on experience

How should stochastics be taught in school? Results of the present study suggest that student-based activity-oriented education, where children make their own hands-on experiences and systematically solve stochastics and where teachers observe the children, show possible strategies and suitable heuristics, and provide the possibility for pupils to 'newly combine' and 'post-create' things, was more successful than the rather teacher-centred approach based on worksheets and more successful than no specific education. This confirms recommendations constructivist educationists (e.g., Gage & Spiegelhalter, 2016; Kimii, 1985; Martignon & Krauss, 2009). However, it contrasts with earlier findings that activity-oriented education only improves the attitudes towards a topic, but not the actual performance (Johnson, Wardlow, & Frandklin, 1997). Our study talks to this issue, and it is among the first to study student-centred activity-oriented education systematically in a large sample. We found that younger children did profit more from this kind of student-centred activity-oriented

stochastics education than did older children. This implies that the earlier stochastic problem-solving is introduced in curricula, the greater the importance of hands-on experience with the topic.

Why most children need knowledge about graphs and tables when they solve probabilistic tasks

We found a significant correlation between test score in tables and the test score in probability tasks ($p = .006$) as well as between the test score in graphs and the test score in combinatorics tasks ($p = .000$). Between combinatorics test score and the probability test score, correlation is also significant ($p = .020$). Research shows that classifications and counting strategies are often used to solve combinatoric and probability tasks (Alexander & Murphy, 1999; Reeves & Weisberg, 1993). To classificate and count systematically, heurisms like tables and graphs are – especially for children – useful tools to build and test hypotheses as they help children to draw comparisons, to find similarities and differences and to contrast possibilities (Pellegrino & Glaser, 1982; Perret, 2015). A good training of strategies using heurisms like tables and graphs improves childrens' performance in combinatorics tasks (Higgins, 2015; Siegler, 2006).

Why activity-oriented education is particularly suitable for boys

Student-based activity-oriented stochastics education was most effective for boys. The finding that the success of a specific education mode interacts with children's gender is in line with previous results. For instance, it has been demonstrated that gender differences in temperament (e.g., effortful control, affecting the ability to quietly follow school lessons) are most pronounced during the elementary school years (Else-Quest, Hyde, Goldsmith, & van Hulle, 2006). Consequently, using the constructivist learning environment specifically for highly active boys has already been suggested (Shu-Chen & Ipsa, 2000). By definition, activity-oriented education furthers activity – it requires action and interaction, not only with the materials provided, but also socially (Cooperstein & Kocevar-Weidinger, 2004). Note, however, that the success of activity-oriented education is not entirely gender specific: In the present study, girls did also improve considerably after activity-oriented education, and more than after receiving a 'worksheet teaching approach'.

Acknowledgements

References

Alexander, P. A., & Murphy, P. K. (1999). Nurturing the seeds of transfer. A domain-specific perspective. *International Journal of Educational Research*, *31*, 561–576. https://doi.org/10.1016/S0883-0355(99)00024-5

Bos, W., Lankes, E., Prenzel, M., Schwippert, K., Walther, G., & Valtin, R. (2003). *Erste Ergebnisse aus IGLU: Schülerleistungen am Ende der vierten Jahrgangsstufe im internationalen*

Vergleich [First results from IGLU. Achievement in grade 4 on an international comparison]. Münster, Germany: Waxmann.

Bosma, T., & Resing, W. C. M. (2006). Dynamic assessment and a reversal task: A contribution to needs-based assessment. *Educational and Child Psychology*, *23*, 81–98.

Butts, D. P., Hoffman, H. M., & Anderson, M. (1993). Is hands-on experience enough? A study of young children's views of sinking and floating objects. *Journal of Elementary Science Education*, *5*(1), 50–64.

Cooperstein, S., & Kocevar-Weidinger, E. (2004). Beyond active learning: A constructivist approach to learning. *Reference Services Review*, *32*, 141–148. https://doi.org/10.1108/009073204 10537658

Curriculum Mathematics for Primary School in Austria. (2007). Retrieved from https://bildung.bmb wf.gv.at/schulen/unterricht/lp/VS7T_Mathematik_3996.pdf?5i81op

Curriculum Mathematics for Primary School in Bavaria. (2014). Retrieved from http://www.lehrpla nplus.bayern.de/fachprofil/grundschule/mathematik

Curriculum Mathematics for Secondary School in Austria. (2014). Retrieved from https://bildung.b mbwf.gv.at/schulen/unterricht/lp/lp_neu_ahs_07_11859.pdf

Curriculum Mathematics for Secondary School in Bavaria. (2014). Retrieved from http://www.le hrplanplus.bayern.de/fachprofil/gymnasium/mathematik

Elliott, J. G., Grigorenko, E. L., & Resing, W. C. M. (2010). Dynamic assessment. In: P. Peterson, E. Baker, & B. McGaw (Eds.), *International encyclopedia of education* (Vol. 3, pp. 220–225). Oxford, UK: Elsevier. https://doi.org/10.1016/B978-0-08-044894-7.00311-0

Else-Quest, N. M., Hyde, J. S., Goldsmith, H. H., & van Hulle, C. A. (2006). Gender differences in temperament: A meta-analysis. *Psychological Bulletin*, *132*, 33–72. https://doi.org/10.1037/ 0033-2909.132.1.33

Gage, J., & Spiegelhalter, D. (2016). *Teaching probability*. Cambridge, UK: Cambridge Press.

Grigorenko, E. L., & Sternberg, R. J. (1998). Dynamic testing. *Psychological Bulletin*, *124*(1), 75–111. https://doi.org/10.1037/0033-2909.124.1.75

Haywood, H. C., & Lidz, C. S. (2007). *Dynamic assessment in practice. Clinical and educational applications*. Cambridge, UK: Cambridge University Press.

Higgins, S. (2015). A recent history of teaching thinking. In R. Wegerif, L. Li, & J. C. Kaufman (Eds.), *The Routledge international handbook of research on teaching thinking* (pp. 19–28). New York, NY: Routledge.

Hodnik-Cadez, T., & Skrbec, M. (2011). Understanding the concepts in probability of pre-school and early school children. *Eurasia Journal of Mathematics, Science & Technology Education*, *7*, 263–279. https://doi.org/10.12973/ejmste/75203

Johnson, D. M., Wardlow, G. W., & Frandklin, T. D. (1997). Hands-on activities versus worksheets in reinforcing physical science principles: Effects on student achievement and attitude. *Journal of Agricultural Education*, *38*, 9–17. https://doi.org/10.5032/jae.1997.03009

Kimii, C. (1985). Leading primary education toward excellence: Beyond worksheets and drill. *Young Children*, *40*, 3–9.

Kipman, U., Fritz, A., Pletzer, B., & Kühberger, A. (2014). Predictors of stochastics achievement in primary school pupils. *Online Educational Research Journal*, *4*, 1–18.

Kohnstamm, G. A. (2014). *Jean Piaget: Children and the inclusion problem*. Livingston, NJ: Transaction Publishers.

Langrall, C. W., & Mooney, E. S. (2005). Characteristics of elementary school students probabilistic reasoning. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 95–119). New York, NY: Springer. https://doi.org/10.1007/b105829

Lehner, S., & Mehltretter, K. (2009). *Kinder entdecken Stochastik. Daten, Wahrscheinlichkeit und Kombinatorik [Children discover stochastics. Data, probability and combinatorics]*. Berlin, Germany: Oldenbourg.

Martignon, L., & Krauss, S. (2009). Hands on activities with fourth-graders: A tool box of heuristics for decision making and reckoning with risk. *International Electronic Journal of Mathematics Education*, *4*(3), 117–148.

Mousalides, N. G., & English, L. D. (2009). Kindergarten students' understanding of the probability concepts. In: M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.). *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education 4* (pp. 137–144). Thessaloniki, Greece: PME.

National Council of Teachers of Mathematics. (2000). *Curriculum and evaluation standards for school mathematics*. Reston, VA: The Council.

Nikiforidou, Z., & Pange, J. (2007). *Sample space and the structure of probability combinations in preschoolers*. Proceedings of CERME 5 (pp. 782–786), Cyprus.

Nikiforidou, Z., & Pange, J. (2009). *Does the nature and amount of posterior information affect preschoolers inferences*. Proceedings of CERME 6 (pp. 388–393), France.

Paparistodemou, E., & Philipou, G. (2002). Intuitions of the concept of probability in 6–7 year-old children. *Themes in Education*, *3*(1), 63–78.

Pellegrino, J., & Glaser, R. (1982). Analyzing aptitudes for learning: Inductive reasoning. *Advances in Instructional Psychology*, *2*, 269–342.

Perret, P. (2015). Children's inductive reasoning: Developmental and educational perspectives. *Journal of Cognitive Education and Psychology*, *14*, 389–408. https://doi.org/10.1891/1945-8959.14.3.389

Piaget, J., & Inhelder, B. (1975). *The origin of the idea of chance in children*. London, UK: Routledge and Kegan Paul; Reston, VA: NCTM.

Reeves, L. M., & Weisberg, R. W. (1993). On the concrete nature of human thinking. Content and context in analogical transfer. *Educational Psychology*, *13*, 245–258. https://doi.org/10.1080/0144341930130303

Resing, W. C. M. (2013). Dynamic testing and individualized instruction. Helpful in cognitive education? *Journal of Cognitive Education and Psychology*, *12*, 81–95. https://doi.org/10.1891/1945-8959.12.1.81

Resing, W. C. M., Tunteler, E., & Elliott, J. G. (2015). The effect of dynamic testing with electronic prompts and scaffolds on children's inductive reasoning. A microgenetic study. *Journal of Cognitive Education and Psychology*, *14*, 231–251. https://doi.org/10.1891/1945-8959.14.2.231

Savery, J., & Duffy, T. (1995). Problem based learning: An instructional model and its constructivist framework. *Educational Technology*, *35*, 31–38.

Schneider, K. (2011). *Handlungsorientierter Unterricht und ein systemisch-konstruktivistisches Verständnis von Lehren und Lernen [Activity-oriented education and a constructivist understanding of learning and teaching]* (pp. 117–127). Schriftenreihe: Pädagogische Materialien der Technischen Universität Kaiserslautern [Publication series at the University of Kaiserslautern], 36. Kaiserslautern, Germany: University of Kaiserslautern.

Shu-Chen, Y., & Ipsa, J. M. (2000). Children's temperament and behavior in Montessori and constructivist early childhood programs. *Early Education and Development*, *11*, 171–186.

Siegler, R. S. (2006). Microgenetic analysis of learning. In W. Damon & R. M. Lerner (Series Eds.) & D. Kuhn, & R. S. Siegler (Vol. Eds.), *Handbook of child psychology: Vol 2. Cognition, perception, and language* (6th ed., pp. 464–510). Hoboken, NJ: John Wiley.

Taber, K. S. (2011). Constructivism as educational theory: Contingency in learning, and optimally guided instruction. In J. Hassaskhah (Ed.), *Educational theory* (pp. 39–61). New York, NY: Nova.

Tunteler, E., Pronk, C. M. E., & Resing, W. C. M. (2008). Inter- and intra-individual variability in the process of change in the use of analogical strategies to solve geometric tasks in children. A microgenetic analysis. *Learning and Individual Differences*, *18*(1), 44–60. https://doi.org/10.1016/j.lindif.2007.07.007

Appendix

Test at the beginning:
  http://www.youtube.com/watch?v=_v7wYUM_oSw
  Definitions 1:
  http://www.youtube.com/watch?v=koXP6zj0pB8
  Definitions 2:
  http://www.youtube.com/watch?v=0pAXsyt-kvQ
  Tables and graphs 1:
  http://www.youtube.com/watch?v=8I6AdaNyiEU
  Tables and graphs 2:
  http://www.youtube.com/watch?v=tDhNZPsGIIw
  Combinatorics 1:
  http://www.youtube.com/watch?v=76QuvTa-cWw
  Combinatorics 2:
  http://www.youtube.com/watch?v=tmyFo7x0pQ8
  Combinatorics 3:
  http://www.youtube.com/watch?v=Wns0iCoWj8U
  Combinatorics 4:
  http://www.youtube.com/watch?v=jJqjgYr5SII
  Combinatorics 5:
  http://www.youtube.com/watch?v=429m5p64JHk
  Random experiments 1:
  http://www.youtube.com/watch?v=TlAd0lc-iVc
  Random experiments 2:
  http://www.youtube.com/watch?v=rlAJJXPwUEk
  Random experiments 3:
  http://www.youtube.com/watch?v=FbXRQ3yIbN0
  Random experiments 4, playing lesson and test at the end:
  http://www.youtube.com/watch?v=MwbW5dQnEWI.